THE HAUSDORFF DIMENSION OF AN EXCEPTIONAL SET IN DIOPHANTINE APPROXIMATION

Abstract. An important consequence of Dirichlet's theorem (1842) is that any irrational number can be approximated by infinitely many rationals with an error of approximation one over denominator square. Surprisingly, most of the metrical results to date improve this corollary instead of Dirichlet's original theorem. Recently, metric theoretic results for the sets of real numbers admitting improvements to Dirichlet's original theorem have been developed by Hussain, Kleinbock and others (2017, 2018). It was noticed that the classical set of well approximable numbers $\mathcal{K}(\Psi)$ is properly contained in the set of Dirichlet non-improvable numbers $G(\Psi)$. In this talk, we will explain how we calculate the Hausdorff dimension for the set $G(\Psi) \setminus \mathcal{K}(\Psi)$ which turns out to be

$$\dim_{\mathrm{H}} \Big(G(\Psi) \setminus \mathcal{K}(\Psi) \Big) = \frac{2}{\tau + 2}, \text{ where } \tau = \liminf_{q \to \infty} \frac{\log \Psi(q)}{\log q}.$$

where $\Psi : [t_0, \infty) \to \mathbb{R}_+$ be a non-decreasing function with $t_0 \gg 1$ fixed. In other words the set of real numbers in $G(\Psi) \setminus \mathcal{K}(\Psi)$ is uncountable. This is a joint work with Mumtaz Hussain.